

# MEASUREMENT OF THE THERMAL DIFFUSIVITY COEFFICIENT OF MATERIALS

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*A theoretical substantiation of the measuring procedure for the thermal diffusivity coefficient of substances is given. In doing this, the two-dimensional problem of heat conduction for a cylindrical semi-infinite specimen and the circular modulated heat flux is considered. The coefficient sought is estimated from the phase shift between temperature fluctuations on the flat surface of the specimen exposed to the heat flux and the fluctuations of this flux.*

Among the possible procedures of measuring thermophysical properties of materials by a method of periodic heating, attention should be paid to the procedure with which the sought thermophysical characteristics of a specimen can be assessed by the corresponding parameters of the temperature wave on its surface exposed to heat.

Thus, for example, the work [1], on the basis of consideration of a one-dimensional heat conduction problem, analyzes the possibility of determining thermal activity of material from a measured temperature wave amplitude on the specimen surface. To reduce the overall error it would be more preferable to measure this wave's phase; however, with heat transfer on the specimen surface, it is impossible to estimate the thermophysical characteristics of its substance from these measurements. Thus, for example, it can be shown that from the phase shift between temperature fluctuations on an infinite flat surface exposed to a one-dimensional heat flux and those of this flux itself, the thermal diffusivity coefficient cannot be uniquely determined due to the presence of the unknown heat-transfer parameter.

Such uniqueness is possible if a two-dimensional temperature wave in the specimen occurs.

The aim of the present work is a theoretical substantiation of a suitable measuring procedure.

We will consider the problem shown in Fig. 1. On the center of the specimen, which is a semiinfinite cylinder placed into the chamber with inertial gas, let there be incident a modulated heat flux with surface density  $q = q \exp(i\omega t)$ , where  $t$  is the time. The coordinate system is shown in the figure. The mathematical problem will then reduce to the solution of the two-dimensional heat conduction equation

$$a \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \frac{\partial^2 T}{\partial z^2} \right) = \frac{\partial T}{\partial t} \quad (1)$$

with the boundary conditions

$$\begin{aligned} z = 0: & -\lambda \frac{\partial T}{\partial z} + \varepsilon \delta \gamma (T^4 - T_1^4) + \lambda_c \frac{T - T_1}{L_1} = \begin{cases} q, & 0 \leq r \leq b, \\ 0, & b < r \leq R, \end{cases} \\ z = \infty: & T = \text{const}(t), \\ r = R: & \lambda \frac{\partial T}{\partial r} + \varepsilon \delta \gamma (T^4 - T_2^4) + \lambda_c \frac{T - T_2}{L_2} = 0, \end{aligned}$$

where  $\varepsilon$ ,  $\delta$ , and  $\gamma$  are the Stefan-Boltzmann constant, the reduced emissivity factor, and the irradiance coefficient;  $\lambda_c$  is the effective thermal conductivity coefficient of the inertial gas;  $T_1$  and  $T_2$  are the temperatures of the chamber walls;  $L_1$  and  $L_2$  are the distances from the walls to the specimen.

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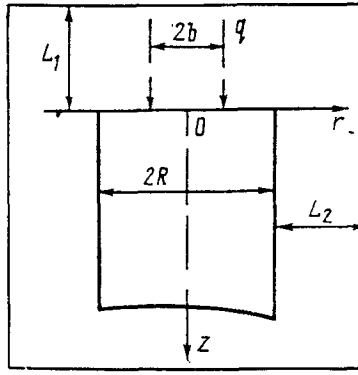


Fig. 1. Statement of the problem.

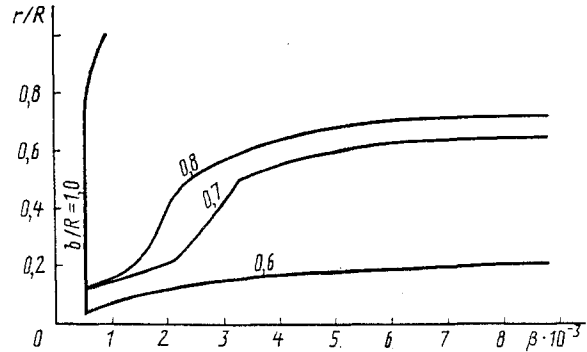


Fig. 2. Boundaries of the regions  $r/R(\beta)$  at different values of  $b/R$ , in which  $|\varphi(r/R) - \varphi(0)|/\varphi(0) < 0.01$  is valid, for  $R = 0.005$ ,  $\alpha_1 = \alpha_2 = \alpha = 12$ ,  $\beta$ ,  $m^{-1}$ .

The solution of this problem under the assumption that for the temperature wave in the specimen  $T(r, z, t) = T(r, z) + \Theta(r, z) \exp(i\omega t)$   $\Theta(r, z) \ll T(r, z)$  is valid, where  $T$  and  $\Theta$  are respectively the constant and variable components of the temperature wave, results in an expression for  $\Theta(r, z)$ , from which it is easy to obtain, for example, for the phase shift

$$\varphi = \text{arctg} \frac{\text{Im} \Theta(r, 0)}{\text{Re} \Theta(r, 0)} = \text{arctg} \frac{\sum_{n=1}^{\infty} \frac{-D_n B_n}{B_n^2 + (A_n + \alpha_1)^2}}{\sum_{n=1}^{\infty} \frac{D_n (A_n + \alpha_1)}{B_n^2 + (A_n + \alpha_1)^2}}, \quad (2)$$

where

$$A_n = \pm \sqrt{(\mu_n^2 \pm \sqrt{\mu_n^4 + \beta^4})/2}; \quad \beta^2 = \omega/a;$$

$$B_n = \pm \sqrt{(-\mu_n^2 \pm \sqrt{\mu_n^4 + \beta^4})/2}; \quad \alpha_1 = \frac{4\epsilon\delta\gamma\bar{T}_0^3}{\lambda} + \frac{\lambda_c}{\lambda L_1};$$

$$D_n = \frac{J_1(\mu_n b) J_0(\mu_n r) \mu_n}{J_0^2(\mu_n R) (\mu_n^2 + \alpha_2^2)}; \quad \alpha_2 = \frac{4\epsilon\delta\gamma\bar{T}_R^3}{\lambda} + \frac{\lambda_c}{\lambda L_2};$$

$J_0$  and  $J_1$  are Bessel functions;  $\bar{T}_0$  and  $\bar{T}_R$  are the mean temperatures of the flat and lateral specimen surfaces;  $\mu_n$  is found from the equation

$$\mu_n J_1(\mu_n R) - \alpha_2 J_0(\mu_n R) = 0.$$

Given the geometric parameters of the problem, expression (2) is the dependence  $\varphi(\omega, a, \alpha_1, \alpha_2)$ . Therefore, in the general case it is sufficient to perform several measurements of  $\varphi$  on different modulation frequencies  $\omega$  in order to determine the sought thermal diffusivity coefficient from the obtained system of equations.

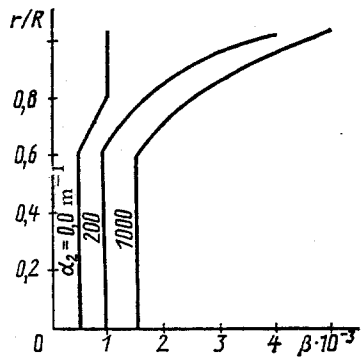


Fig. 3. Boundaries of the regions  $r/R(\beta)$  at different values of  $\alpha_2$  in which  $|\varphi(\alpha_2) - \varphi(12)|/\varphi(12) < 0.01$ , for  $\alpha_1 = 12$ ,  $b/R = 0.8$ ,  $R = 0.005$ .

In order to decrease the required number of equations and, hence, the amount of necessary measurements and to simplify the equations themselves, an analysis of the behavior of the solution obtained was carried out on a computer, some results of which are presented in Figs. 2 and 3.

Figure 2 shows the isophase regions of the temperature wave on the flat specimen surface for the heat-transfer parameter specified. To the right of the indicated boundary lines, when performing measurements, one may disregard the influence of deviation from the central surface point with an error of 1%.

Figure 3 illustrates the influence of heat transfer on the two surfaces of the specimen on the results of measurements. To the right of the indicated boundary lines, when performing measurements, one may disregard the difference between the value of heat transfer on the lateral surface and that on the flat surface with an error of 1%.

Thus, the method of measuring the thermal diffusivity coefficient [2], including the exposure of the specimen surface to the modulated heat flux, the recording of the phase shift between temperature fluctuations on this surface and fluctuations of the flux itself at least at two frequencies of its modulation, and the calculation of the sought coefficient with allowance for the results of the present work, is realized in the relevant measuring procedure which can also be expanded to measurements of other thermophysical characteristics, for example, heat capacity.

In conclusion it should be pointed out that the described method of measurements can be applied to the investigation of thermophysical properties of liquid materials though in considering the problem no account was taken of convective processes which may occur in a liquid substance. With some margin the boundary of hydrodynamic stability can be determined from the condition [3]:  $\partial T/\partial z \leq 0$  (if the  $z$ -axis is co-directed with the gravity axis), from which we can obtain

$$\sqrt{\operatorname{Re}^2 \Theta(r, z) + \operatorname{Im}^2 \Theta(r, z)} \leq \sqrt{\frac{2a}{\omega} \frac{\partial \bar{T}(r, z)}{\partial z}}.$$

When this condition is met the convective process will not affect the results of the measurements performed.

## CONCLUSION

1. The thermal diffusivity coefficient can be determined by solving a system of equations of the form (2) obtained from the corresponding number of measurements of the phase shift between temperature fluctuations on the specimen surface exposed to the heat flux and this flux's fluctuations at different modulation frequencies.

2. This number of equations and hence of measurements can be decreased and the equations themselves be simplified on the basis of results of analyzing the influence of various parameters of the problem considered.

## NOTATION

$\lambda$ , thermal conductivity coefficient of the specimen,  $W/(m \cdot K)$ ;  $a$ , thermal diffusivity coefficient of the specimen,

$m^2/\text{sec}$ ; T, specimen temperature, K; R, specimen radius, m; b, radius of the heat flux on the exposure surface, m;  $q_0$ , heat flux density amplitude,  $W/m^2$ ;  $\omega$ , modulation frequency of the heat flux, rad/sec.

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## INFLUENCE OF PARTICLE COMPACTION ON THERMOPHYSICAL PROPERTIES OF A LAYER

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*Problems of compaction of a layer of deposited particles and its influence on the thermophysical parameters of the layer are considered.*

Most technological processes, performed with disperse systems, are accompanied by deposition of the solid phase on the heat-and-mass-exchange surface, which substantially affects the coefficients of heat, mass, and momentum transfer [1, 2]. Deposition of particles of various kinds on the surface is determined by many factors, among which it is important to distinguish hydrodynamic and thermodynamic conditions, rheological properties of the disperse system, adhesive compatibility of particles with the surface in flow, physicochemical transformations in the boundary layer, etc. At the same time the layer formed by particle deposition is subject to deformation and compaction under the action of external deforming stresses, which substantially affects the heat transfer through a two-layer wall and the thermophysical properties of the deposited layer proper. The mechanism of compacting the layer of disperse particles may be classified in the following manner: a) the dislocation mechanism, whose essence consists of redislocation of particles in elastobound systems under the action of external loads; b) mechanical compaction due to the action of external deforming stresses and mass forces [3]; c) compaction by diffusion of small particles under the action of diffusion thermophoresis [4]; d) the capillary mechanism of compaction in the presence of phase transformations (melting, dissolution, etc.) [4].

One-dimensional compaction of particles in the layer under the action of the external deforming stresses  $\sigma_D$  may be represented by the equation of porosity variation in the form [3]

$$\frac{d\Theta}{dt} = -(1 - \Theta) \eta_s^{-1} \sigma_D. \quad (1)$$

Expressing the bulk viscosity  $\xi_s$  in terms of the shear one and changing from the substantial derivative to the local ones, we obtain

$$\frac{\partial\Theta}{\partial t} + V_s \text{grad } \Theta = -\frac{3}{4} \Theta \eta_s^{-1} \sigma_D, \quad (2)$$

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